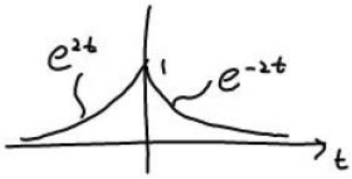
1.1

(a) $x(t) = e^{-2|t|} = \begin{cases} e^{-2t} & t \geq 0 \\ e^{2t} & t < 0 \end{cases}$

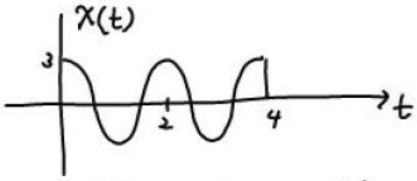


$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\int_0^{\infty} e^{-4t} dt$$

$$= \left[-\frac{1}{2} e^{-4t}\right]_0^{\infty} = \frac{1}{2}$$

$0 < E < \infty$ 이므로 에너지 신호

(b)



$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^4 9\cos^2 \pi t \, dt = \int_0^4 \frac{9(1+\cos 2\pi t)}{2} dt = 18$$

$0 < E < \infty$ 이므로 에너지 신호

1.2

(e) $T = 4$, $\omega_0 = \frac{\pi}{2}$

$$x_{dc} = \frac{1}{4}\int_0^4 x(t)\, dt = 0$$

$$P = \frac{1}{4}\int_0^4 x^2(t)\, dt = \frac{1}{4} \cdot 4\int_0^1 t^2 dt = \frac{1}{3}$$

$$x_{rms} = \frac{1}{\sqrt{3}}$$

(h) $T = 2$, $\omega_0 = \pi$

$$x_{dc} = \frac{1}{2}\int_0^1 \sin \pi t \, dt = \frac{1}{2}\left[-\frac{1}{\pi}\cos \pi t\right]_0^1 = \frac{1}{\pi}$$

$$P = \frac{1}{2}\int_0^1 \sin^2 \pi t \, dt = \frac{1}{2}\int_0^1 \left\{\frac{1-\cos 2\pi t}{2}\right\} dt = \frac{1}{4}$$

$$x_{rms} = \sqrt{P} = \frac{1}{2}$$

1.3

(a) $y(t) = 2t\,x(t)$

i) linearity

$y_1 = T[x_1] = 2t\,x_1$, $y_2 = T[x_2] = 2t\,x_2$

$T[\alpha x_1 + \beta x_2] = 2t(\alpha x_1 + \beta x_2)$

$= \alpha(2t\,x_1) + \beta(2t\,x_2)$

$= \alpha T[x_1] + \beta T[x_2] = \alpha y_1 + \beta y_2$

$\Rightarrow$ linear

ii) time invariance

Let $y_1 = T[x_1] = 2t\,x_1(t)$

Let $x_2 = x_1(t-t_0)$

Then $y_2 = T[x_2] = 2t\,x_2(t) = 2t\,x_1(t-t_0)$

$\neq y_1(t-t_0) = 2(t-t_0)\,x_1(t-t_0)$

$\Rightarrow$ time varying

iii) 인과성

time t에서의 출력은 time t에서의 입력에만 관계됨

$\Rightarrow$ causal

iv) stability

$x(t) = 1$ 이라 하자 $\Rightarrow |x(t)| = 1 < \infty$ for all t

$y(t) = 2t\,x(t) = 2t \Rightarrow |y(t)| = 2|t| \to \infty$ as $t \to \infty$

$\Rightarrow$ unstable

(e) $y(t) = x(t)\,x(t-1)$

i) linearity

Let $x(t) = 1$, then $x(t-1) = 1$

$$x(t) \longrightarrow \boxed{T(\cdot)} \longrightarrow y(t) = 1 \cdot 1 = 1$$

For $x_1(t) = 2x(t)$, $x_1(t) = 2$, $x_1(t-1) = 2$

$$\underset{= 2x(t)}{x_1(t)} \longrightarrow \boxed{T(\cdot)} \longrightarrow y_1(t) = T[x_1(t)] = x_1(t)\,x_1(t-1)$$

$$= 2 \cdot 2 = 4$$

$$\neq 2y(t)$$

$\Rightarrow$ nonlinear

ii) time invariance

$$x(t) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(t) = x(t)\,x(t-1)$$

shift $\downarrow t_0$

$$x_1(t) = x(t-t_0) \longrightarrow \boxed{T[\cdot]} \longrightarrow y_1(t) = T[x_1(t)] = x_1(t-t_0)\,x_1(t-t_0-1)$$

$$= y(t-t_0)$$

$\Rightarrow$ time invariant

iii) causality

t에서의 출력은 time t 와 t-1 (과거)의 입력에만 영향을 받으므로 causal

iv) Stability

$|x(t)| \le B < \infty \quad \forall t$ 라고 가정하자.

$|y(t)| = |x(t)| \cdot |x(t-1)| \le B^2 < \infty$ 이므로 Stable