6.9

(a) $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-2)^2}{18}}$

$m=2,\ \sigma=3,\ \sigma^2=9$

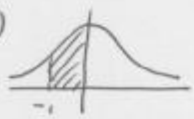
(b) $Y = \frac{X-m}{\sigma} = \frac{X-2}{3}$ 로 정의하면

Y는 평균이 0 이고 분산이 1 인 Gaussian 확률변수가 된다.

$P(X>3) = P(Y>\frac{1}{3}) = \int_{\frac{1}{3}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy = Q(\frac{1}{3})$

(c) $P(-1 \le X < 2) = P(X \geq -1) - P(X \geq 2)$

$= P(Y \geq -1) - P(Y \geq 0) = Q(-1) - Q(0)$



$= Q(-1) - \frac{1}{2}$

$= \{1 - Q(1)\} - \frac{1}{2} = \frac{1}{2} - Q(1)$

(d) $P(X<4) = 1 - P(X \geq 4) = 1 - P(Y \geq \frac{2}{3})$

$= 1 - Q(\frac{2}{3})$

6.17

$E[Z] = E[2X+3Y] = 2E[X] + 3E[Y] = 2m_X + 3m_Y$

$= 2\times 2 + 3\times 1 = 7$

$\sigma_Z^2 = E[(Z-m_Z)^2] = E[(2X+3Y-2m_X-3m_Y)^2]$

$= E[\{2(X-m_X) + 3(Y-m_Y)\}^2]$

$= E[4(X-m_X)^2 + 12(X-m_X)(Y-m_Y) + 9(Y-m_Y)^2]$

$= 4\sigma_X^2 + 9\sigma_Y^2 + 12\sigma_X\sigma_Y\rho_{XY}$

$= 4\times 4 + 9\times 8 + 12\times 2\times 2\sqrt{2}\,\rho_{XY} = 88 + 48\sqrt{2}\,\rho_{XY}$

(a) $\rho_{XY} = 0 \Rightarrow \sigma_Z^2 = 88$

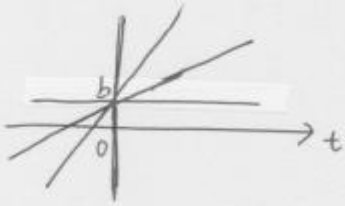
(b) $\rho_{XY} = \frac{1}{4} \Rightarrow \sigma_Z^2 = 88 + 48\sqrt{2}\times\frac{1}{4} = 88 + 12\sqrt{2} = 104.97$

(c) $\rho_{XY} = \frac{1}{2} \Rightarrow \sigma_Z^2 = 88 + 24\sqrt{2} = 121.94$

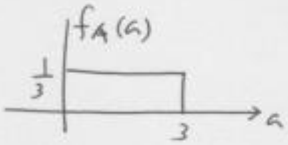
(d) $\rho = 1 \Rightarrow \sigma_Z^2 = 88 + 48\sqrt{2} = 155.88$

6.20

(a) $X(t) = At + b$



(b) $E[X(t)] = E[At+b] = E[A]t + b$



$$E[A] = \frac{3}{2}$$

$$E[A^2] = \int_{-\infty}^{\infty} a^2 f_A(a) da$$

$$= \int_0^3 a^2 \cdot \frac{1}{3} da = 3$$

따라서 $E[X(t)] = \frac{3}{2}t + b$

(c) $R_X(t_1, t_2) = E[X(t_1)X(t_2)]$

$$= E[(At_1 + b)(At_2 + b)]$$

$$= E[A^2 t_1 t_2 + Abt_1 + Abt_2 + b^2]$$

$$= E[A^2]t_1 t_2 + t_1 E[A]b + t_2 E[A]b + b^2$$

$$= 3t_1 t_2 + \frac{3}{2}(t_1 + t_2)b + b^2$$

(d) $R_X(t_1, t_2) \neq R_X(t_1 - t_2)$, $E[X(t)] \neq$ 상수

따라서 WSS 가 아님.

6.33

예제 6.9로부터 $M(t)$가 기저대역 프로세스일 때

$$X(t) = M(t)\cos(\omega_0 t + \Theta), \quad Y(t) = M(t)\sin(\omega_0 t + \Theta)$$ 는

다음과 같은 PSD를 갖는다:

$$S_X(f) = S_Y(f) = \frac{1}{4}\left[S_M(f-f_0) + S_M(f+f_0)\right]$$

따라서 문제 6.33에서

$$X(t) = \underbrace{N(t)\cos(2\pi f_0 t + \Theta)}_{X_1(t)} + \underbrace{N(t)\sin(2\pi f_0 t + \Theta)}_{X_2(t)}$$

$X_1(t)$와 $X_2(t)$는 모두 동일한 PSD

$$\frac{1}{4}\left[S_n(f-f_0) + S_n(f+f_0)\right]$$

를 갖는다. 그러므로 $X(t)$의 PSD는 다음과 같다.

$$S_X(f) = \frac{1}{2}\left[S_n(f-f_0) + S_n(f+f_0)\right]$$